

On the energy of particle collisions in the ergosphere of the rotating black holes

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PACS 04.70.-s – Physics of black holes

PACS 04.70.Bw – Classical black holes

PACS 97.60.Lf – Black holes

Abstract – It is shown that the energy in the centre of mass frame of colliding particles in free fall at any point of the ergosphere of the rotating black hole can grow without limit for fixed energy values on infinity. The effect takes place for large negative values of the angular momentum of one of the particles.

Introduction. – Recently many papers were published [1]– [10] in which some specific properties of collisions of particles close to the horizon of the rotating black holes were discussed. In [1] the effect of unlimited growing energy of colliding two particles in the centre of mass frame for critical Kerr rotating black holes (call it BSW effect) was discovered. In our papers [2]– [4] the same effect was found for noncritical black holes when multiple collisions took place.

All this shows that close to the horizon there is natural supercollider of particles with the energies up to Planck's scale. However its location close to the horizon makes difficult due to the large red shift get some observed effects outside of the ergosphere when particles go from the “black hole supercollider” outside. Here we shall discuss some other effect valid at any point of the ergosphere. The energy in the centre of mass frame can take large values for large negative values of the angular momentum projection on the rotation axis of the black hole. The problem is how to get this large (in absolute value) negative values. That is why first we shall obtain limitations on the values of the projection of angular momentum outside ergosphere for particles falling into the black hole and inside the ergosphere. It occurs that inside the ergosphere of

black hole there is no limit for negative values of the momentum and arbitrary large energy in the centre of mass frame is possible for large angular momentum of the two colliding particles. In a sense the effect is similar to the BSW-effect. One of the particle with large negative angular momentum can be called “critical”, the other particle “ordinary”.

The system of units $G = c = 1$ is used.

General formulas for the collision energy close to the black hole. – The Kerr's metric of the rotating black hole [11] in Boyer–Lindquist coordinates [12] has the form:

$$ds^2 = dt^2 - \frac{2Mr}{\rho^2} (dt - a \sin^2 \theta d\varphi)^2 - \rho^2 \left(\frac{dr^2}{\Delta} + d\theta^2 \right) - (r^2 + a^2) \sin^2 \theta d\varphi^2, \quad (1)$$

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2, \quad (2)$$

M is the mass of the black hole, aM its angular momentum. The rotation axis direction corresponds to $\theta = 0$, i.e. $a \geq 0$. The event horizon of the Kerr's black hole corresponds to

$$r = r_H \equiv M + \sqrt{M^2 - a^2}. \quad (3)$$

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The surface of the static limit is defined by

$$r = r_1(\theta) \equiv M + \sqrt{M^2 - a^2 \cos^2 \theta}. \quad (4)$$

In case $a \leq M$ the region of space-time between the static limit and event horizon is called ergosphere.

For geodesics in Kerr's metric (1) one obtains (see [13], Sec. 62 or [14], Sec. 3.4.1)

$$\rho^2 \frac{dt}{d\lambda} = -a(aE \sin^2 \theta - J) + \frac{r^2 + a^2}{\Delta} P, \quad (5)$$

$$\rho^2 \frac{d\varphi}{d\lambda} = -\left(aE - \frac{J}{\sin^2 \theta}\right) + \frac{aP}{\Delta}, \quad (6)$$

$$\rho^2 \frac{dr}{d\lambda} = \sigma_r \sqrt{R}, \quad \rho^2 \frac{d\theta}{d\lambda} = \sigma_\theta \sqrt{\Theta}, \quad (7)$$

$$P = (r^2 + a^2)E - aJ, \quad (8)$$

$$R = P^2 - \Delta[m^2 r^2 + (J - aE)^2 + Q], \quad (9)$$

$$\Theta = Q - \cos^2 \theta \left[a^2(m^2 - E^2) + \frac{J^2}{\sin^2 \theta} \right]. \quad (10)$$

Here E is conserved energy (relative to infinity) of the probe particle, J is conserved angular momentum projection on the rotation axis of the black hole, m is the rest mass of the probe particle, for particles with nonzero rest mass $\lambda = \tau/m$, where τ is the proper time for massive particle, Q is the Carter's constant. The constants σ_r, σ_θ in formulas (7) are equal ± 1 and are defined by the direction of particle movement in coordinates r, θ . For massless particles one must take $m = 0$ in (9), (10).

One can find the energy in the centre of mass frame of two colliding particles $E_{c.m.}$ with rest masses m_1, m_2 taking the square of

$$(E_{c.m.}, 0, 0, 0) = p_{(1)}^i + p_{(2)}^i, \quad (11)$$

where $p_{(n)}^i$ are 4-momenta of particles ($n = 1, 2$). Due to $p_{(n)}^i p_{(n)i} = m_n^2$ one has

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2p_{(1)}^i p_{(2)i}. \quad (12)$$

Note that the energy of collisions of particles in the centre of mass frame is always positive (while the energy of one particle due to Penrose effect [15] can be negative!) and satisfies the condition

$$E_{c.m.} \geq m_1 + m_2. \quad (13)$$

This follows from the fact that the colliding particles move one towards another with some velocities.

It is important to note that $E_{c.m.}$ for two colliding particles is not a conserved value differently from energies of particles (relative to infinity) E_1, E_2 .

For the free falling particles with energies E_1, E_2 and angular momentum projections J_1, J_2 from (5)–(9) one obtains [5]:

$$E_{c.m.}^2 = m_1^2 + m_2^2 + \frac{2}{\rho^2} \left[\frac{P_1 P_2 - \sigma_{1r} \sqrt{R_1} \sigma_{2r} \sqrt{R_2}}{\Delta} - \frac{(J_1 - aE_1 \sin^2 \theta)(J_2 - aE_2 \sin^2 \theta)}{\sin^2 \theta} - \sigma_{1\theta} \sqrt{\Theta_1} \sigma_{2\theta} \sqrt{\Theta_2} \right]. \quad (14)$$

Limitations on the values of particle angular momentum close to the Kerr's black hole. – The permitted region for particle movement is defined by conditions

$$\Theta = Q - \cos^2 \theta \left[a^2(m^2 - E^2) + \frac{J^2}{\sin^2 \theta} \right] \geq 0, \quad (15)$$

$R \geq 0$, and movement “forward in time” leads to $dt/d\lambda > 0$ [16]. The condition (15) gives possible values of Carter's constant Q . From (7) it follows that for movement with constant θ it is necessary and sufficient that $\Theta = 0$. It is always possible to choose this value, so that for geodesics in the equatorial plane ($\theta = \pi/2$) Carter's constants $Q = 0$.

Let us find limitations for the particle angular momentum from the conditions $R \geq 0, dt/d\lambda > 0$ at the point (r, θ) , taking the fixed values of Θ . Outside the ergosphere $r^2 - 2rM + a^2 \cos^2 \theta > 0$ one obtains

$$E \geq \frac{1}{\rho^2} \sqrt{(m^2 \rho^2 + \Theta)(r^2 - 2rM + a^2 \cos^2 \theta)}, \quad (16)$$

$$J \in [J_-, J_+], \quad (17)$$

$$J_{\pm} = \frac{\sin \theta}{r^2 - 2rM + a^2 \cos^2 \theta} \left[-2rMaE \sin \theta \pm \sqrt{\Delta(\rho^4 E^2 - (m^2 \rho^2 + \Theta)(r^2 - 2rM + a^2 \cos^2 \theta))} \right]. \quad (18)$$

Boundary values of J_{\pm} correspond to $dr/d\lambda = 0$.

On the boundary of ergosphere

$$r = r_1(\theta) \Rightarrow E \geq 0, \quad (19)$$

$$J \leq E \left[\frac{Mr_1(\theta)}{a} + a \sin^2 \theta \left(1 - \frac{m^2}{2E^2} - \frac{\Theta}{4Mr_1(\theta)E^2} \right) \right]. \quad (20)$$

Inside ergosphere

$$r_H < r < r_1(\theta) \Rightarrow (r^2 - 2rM + a^2 \cos^2 \theta) < 0, \quad (21)$$

$$J \leq J_-(r, \theta) = \frac{\sin \theta}{-(r^2 - 2rM + a^2 \cos^2 \theta)} \left[2rMaE \sin \theta - \sqrt{\Delta(\rho^4 E^2 - (m^2 \rho^2 + \Theta)(r^2 - 2rM + a^2 \cos^2 \theta))} \right]. \quad (22)$$

So on the *boundary and inside ergosphere there exist geodesics on which particle with fixed energy can have arbitrary large in absolute value negative angular momentum projection*.

From (22) one can see that for negative energy E of the particle in ergosphere its angular momentum projection on the rotation axis of the black hole must be also negative. This is a well known Penrose effect [15]. However rotation in Boyer–Lindquist coordinates for any particle in ergosphere has the same direction as the rotation of the black hole (the effect of the “dragging” of bodies by the rotating black hole [14]). Really for timelike geodesics $ds^2 > 0$ leads to $d\varphi/dt > 0$. So it is incorrect to say (as it is said in some test books on black holes [13], p. 368) that “only counter-rotating particles can have negative energy”! Inside the ergosphere the usual intuition which is true far outside it that the change of the sign of angular momentum projection on Z -axis means the change of the rotation on counter-rotation following from usual formula $\mathbf{J} = \mathbf{r} \times \mathbf{p}$ is incorrect.

From equations of geodesics (6) one can see the peculiar properties of the correspondence between the direction of rotation and the angular momentum projection. Let us rewrite it as

$$\rho^2 \sin^2 \theta \frac{d\varphi}{d\lambda} = \frac{E2Mr a \sin^2 \theta}{\Delta} + J \frac{r^2 - 2rM + a^2 \cos^2 \theta}{\Delta}. \quad (23)$$

For large r outside the ergosphere one gets the standard expression for the angular momentum projection in Minkowski space $J = mr^2 \sin^2 \theta d\varphi/d\tau$ and $J = mr^2 d\varphi/d\tau$ for $\theta = \pi/2$.

But when one is ingoing inside the ergosphere one sees that the coefficient for J in (23) becomes zero on its surface so that the angular velocity $d\varphi/d\lambda$ and $d\varphi/d\tau$ is defined by the energy and does not depend on J at all.

Inside the ergosphere the coefficient for J in (23) becomes negative, the angular velocity is still positive and one comes to unusual conclusion: *if the energy of the particle in ergosphere is fixed particles with negative angular momentum projection are rotating in the direction of rotation of the black hole with greater angular velocity!*

So the constant characterizing the geodesic which coincides with the usual angular momentum definition taken from Newtonian physics outside the black hole does not coincide with it in ergosphere.

Energy of collision with a particle with large angular momentum. – Let us find the asymptotic of (14) for $J_2 \rightarrow -\infty$ and some fixed value r in ergosphere supposing the value of Carter’s constant Q_2 to be such that (15) is valid and $\Theta_2 \ll J_2^2$. Then from (14) one obtains

$$E_{c.m.}^2 \approx \frac{-2J_2}{\rho^2 \Delta} \left[\frac{J_1}{\sin^2 \theta} (r^2 - 2rM + a^2 \cos^2 \theta) + 2rMaE_1 - \frac{\sigma_{1r}\sigma_{2r}\sqrt{R_1}}{\sin \theta} \sqrt{-(r^2 - 2rM + a^2 \cos^2 \theta)} \right]. \quad (24)$$

This asymptotic formula is valid for all possible E_1, J_1 (see (22)) for $r_H < r < r_1(\theta)$ and for $E_1 > 0$ and J_1 satisfying (20) for $r = r_1(\theta)$. The poles $\theta = 0, \pi$ are not considered here because the points on surface of static limit are on the event horizon.

Note that expression in brackets in (24) is positive in ergosphere. This is evident for $r = r_1(\theta)$ and follows from limitations (22) for $r_H < r < r_1(\theta)$, and inside ergosphere (24) can be written as

$$E_{c.m.}^2 \approx J_2 \frac{r^2 - 2rM + a^2 \cos^2 \theta}{\rho^2 \Delta \sin^2 \theta} \times \left(\sigma_{1r} \sqrt{J_{1+} - J_1} - \sigma_{2r} \sqrt{J_{1-} - J_1} \right)^2. \quad (25)$$

So from (24) one comes to the conclusion that *when particles fall on the rotating black hole collisions with arbitrarily high energy in the centre of mass frame are possible at any point of the ergosphere if $J_2 \rightarrow -\infty$ and the energies E_1, E_2 are fixed*. The energy of collision in the centre of mass frame is growing proportionally to $\sqrt{|J_2|}$.

Note that for large $-J_2$ the collision energy close to horizon in the centre of mass frame depending on values E_1, J_1 can be as large as less then for collisions at the other points of ergosphere.

Outside ergosphere the collision energy is limited for given r , but for $r \rightarrow r_1$ it can be large if one of the particles gets in intermediate collisions the angular momentum close to J_- (see (18)). On fig. 1 the dependence of collision energy in the centre of mass frame on the coordinate r is shown for particles with $E_1 = E_2 = m_1 = m_2$, $J_1 = 0$ and $J_2 = J_-$ moving in equatorial plane of black hole with $a = 0.8M$.

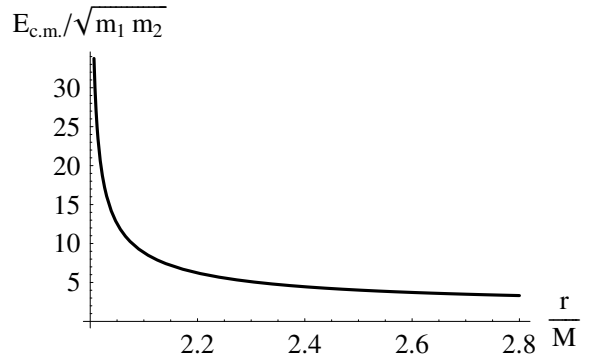


Fig. 1: The collision energy in the centre of mass frame for particles with $J_1 = 0$ and $J_2 = J_-$ out of the ergosphere.

Note that large negative values of the angular momentum projection are forbidden for fixed values of energy of particle out of the ergosphere. So particle which is non-relativistic on space infinity ($E = m$) can arrive to the horizon of the black hole if its angular momentum projection is located in the interval

$$-2mM \left[1 + \sqrt{1 + \frac{a}{M}} \right] \leq J \leq 2mM \left[1 + \sqrt{1 - \frac{a}{M}} \right]. \quad (26)$$

The left boundary is a minimal value of the angular momentum of particles with $E = m$ capable to achieve ergosphere falling from infinity. That is why collisions with $J_2 \rightarrow -\infty$ do not occur for particles following from infinity. But if the particle came to ergosphere and there in the result of interactions with other particles is getting large negative values of the angular momentum projection (no need for getting high energies!) then its subsequent collision with the particle falling on the black hole leads to high energy in the centre of mass frame.

Getting superhigh energies for collision of usual particles (i.e. protons) in such mechanism occur however physically nonrealistic. Really from (24) the value of angular momentum necessary for getting the collision energy $E_{c.m.}$ has the order

$$J_2 \approx -\frac{aE_{c.m.}^2}{2E_1}. \quad (27)$$

So from (26) absolute value of the angular momentum J_2 must acquire the order $E_{c.m.}^2/(m_1m_2)$ relative to the maximal value of the angular momentum of the particle incoming to ergosphere from infinity. For example if $E_1 = E_2 = m_p$ (the proton mass) then $|J_2|$ must increase with a factor 10^{18} for $E_{c.m.} = 10^9 m_p$. To get this one must have very large number of collisions with getting additional negative angular momentum in each collision.

However the situation is different for supermassive particles. In [17]–[19] we discussed the hypothesis that dark matter contains stable superheavy neutral particles with mass of the Grand Unification scale created by gravitation in the end of the inflation era. These particles are nonstable for energies of interaction of the order of Grand Unification and decay on particles of visual matter but are stable at low energies. But in ergosphere of the rotating black holes such particles due to getting large relative velocities can increase their energy from $2m$ to values of $3m$ and larger so that the mechanism considered in our paper can lead to their decays as it was in the early universe. The number of intermediate collisions for them is not very large (of the order of 10).

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The research is supported by the grant from The John Templeton Foundation.

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